

“Finite” Non-Gaussianities and Tensor-Scalar Ratio in Large Volume Swiss-Cheese Compactifications

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Abstract

Developing on the ideas of (section 4 of) [1] and [2] and using the formalisms of [3, 4], after inclusion of perturbative and non-perturbative α' corrections to the Kähler potential and ($D1$ - and $D3$ -) instanton generated superpotential, we show the possibility of getting finite values for the non-linear parameter f_{NL} while looking for non-Gaussianities in type IIB compactifications on orientifolds of the Swiss Cheese Calabi-Yau $\mathbf{WCP}^4[1, 1, 1, 6, 9]$ in the L(arge) V(olume) S(cenarios) limit. First we show that in the context of multi-field slow-roll inflation, for the Calabi-Yau volume $\mathcal{V} \sim 10^5$ and $D3$ -instanton number $n^s \sim 10$ along with $N_e \sim 18$, one can realize $f_{NL} \sim 0.03$, and for Calabi-Yau volume $\mathcal{V} \sim 10^6$ with $D3$ -instanton number $n^s \sim 10$ resulting in number of e-foldings $N_e \sim 60$, one can realize $f_{NL} \sim 0.01$. Further we show that with the slow-roll conditions violated and for the number of the $D3$ -instanton wrappings $n^s \sim \mathcal{O}(1)$, one can realize $f_{NL} \sim \mathcal{O}(1)$. Using general considerations and some algebraic geometric assumptions, we show that with requiring a “freezeout” of curvature perturbations at super horizon scales, it is possible to get tensor-scalar ratio $r \sim \mathcal{O}(10^{-3})$ with the loss of scale invariance $|n_R - 1| = 0.01$ and one can obtain $f_{NL} \sim \mathcal{O}(10^{-2})$ as well in the context of slow-roll inflation scenarios in the same Swiss-Cheese setup. For all our calculations of the world-sheet instanton contributions to the Kähler potential coming from the non-perturbative α' corrections, the degrees of genus-zero rational curves correspond to the largest value of the Gopakumar-Vafa invariants for the chosen compact projective variety, which is very large. To our knowledge, such values of non-Gaussianities and tensor-scalar ratio in slow-roll inflationary and/or slow-roll violating scenarios, have been obtained for the first time from string theory. We also make some observations pertaining to the possibility of the axionic inflaton also being a cold dark matter candidate as well as a quintessence field used for explaining dark energy.

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1 Introduction

The idea of inflation has widely involved several string theorists for the past few years with attempts to constructing realistic inflationary models to have a match with the cosmological observations [5, 6, 7, 8, 9] as a test of string theory [10, 11, 12, 13, 14]. Although the idea of inflation was initially introduced to explain the homogeneous and isotropic nature of the universe at large scale structure [15, 16, 17], its best advantage is reflected while studying inhomogeneities and anisotropies of the universe, which is a consequence of the vacuum fluctuations of the inflaton as well as the metric fluctuations. These fluctuations result in non-linear effects (parametrized by f_{NL}, τ_{NL}) seeding the non-Gaussianity of the primordial curvature perturbation, which are expected to be observed by PLANCK, with non-linear parameter $f_{NL} \sim \mathcal{O}(1)$ [18]. Along with the non-linear parameter f_{NL} , the “tensor-to-scalar ratio” r is also one of the key inflationary observables, which measures the anisotropy arising from the gravity-wave(tensor) perturbations and the signature of the same is expected to be predicted by the PLANCK if the tensor-to-scalar ratio $r \sim 10^{-2} - 10^{-1}$ [18]. As these parameters give a lot of information about the dynamics inside the universe, the theoretical prediction of large/finite (detectable) values of the non-linear parameters f_{NL}, τ_{NL} as well as “tensor-to-scalar ratio” r has received a lot of attention for recent few years [7, 8, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31]. For calculating the non-linear parameter f_{NL} , a very general formalism (called as δN -formalism) was developed and applied for some models [32]. Initially the parameter f_{NL} was found to be suppressed (to undetectable value) by the slow roll parameters in case of the single inflaton model. Followed by this, several models with multi-scalar fields have been proposed but again with the result of the non-linear parameter f_{NL} of the order of the slow-roll parameters as long as the slow-roll conditions are satisfied [3, 4, 5, 33, 34, 35, 36, 37, 38, 39, 40]. Recently considering multi-scalar inflaton models, Yokoyama et al have given a general expression for calculating the non-linear parameter f_{NL} (using δN -formalism) for non-separable potentials[3] and found the same to be suppressed again by the slow-roll parameter ϵ (with an enhancement by exponential of quantities $\sim \mathcal{O}(1)$). In the work followed by the same as a generalization to the non slow-roll cases, the authors have proposed a model for getting finite f_{NL} violating the slow roll conditions temporarily [4]. The observable “tensor-to-scalar ratio” r , characterizing the amount of anisotropy arising from scalar-density perturbations (reflected as the CMB quadrupole anisotropy) as well as the gravity-wave perturbations arising through the tensorial metric fluctuations, is crucial for the study of temperature/angular anisotropy from the CMB observations. The “tensor-to-scalar ratio” r is defined as the ratio of squares of the amplitudes of the tensor to the scalar perturbations defined through their corresponding power spectra. Several efforts have been made for getting large/finite value of “ r ” with different models, some resulting in small undetectable values while some predicting finite bounds for the same [19, 25, 26, 27, 28, 29, 30].

In this note, continuing with the results of our previous paper [2] for Large Volume multi-axionic Swiss-Cheese inflationary scenarios, we discuss whether it is possible, *starting from string theory*, to

- obtain $f_{NL} \sim \mathcal{O}(10^{-2} - 10^0)$,
- obtain tensor-scalar ratio $r \sim 10^{-3}$,
- obtain number of e-foldings $N_e \sim \mathcal{O}(10)$,
- obtain the loss of scale invariance within experimental bound: $|n_R - 1| \leq 0.05$
such that
- the curvature perturbations are “frozen” at super horizon scales,
- the inflaton could be a dark matter candidate at least in some corner of the moduli space,

- the inflaton could also be identified with quintessence to explain dark energy, once again, at least in some corner - the same as above - of the moduli space.

We find that the answer is a yes. The crucial input from algebraic geometry that we need is the fact that Gopakumar-Vafa invariants of genus-zero rational curves for compact Calabi-Yau three-folds expressed as projective varieties can be very large for appropriate maximum values of the degrees of the rational curves. This is utilized when incorporating the non-perturbative α' contribution to the Kähler potential.

The plan of the paper is as follows. In section 2, we review our previous work pertaining to obtaining meta-stable dS vacua without addition of anti-D3 branes and obtaining axionic slow-roll inflation. In section 3, using the techniques of [3, 4], we show the possibility of getting finite $\mathcal{O}(10^{-2})$ non-Gaussianities in slow-roll and $\mathcal{O}(1)$ non-Gaussianities in slow-roll violating scenarios. In section 4, based on general arguments not specific to our (string-theory) set-up and using the techniques of [23, 41], we show that ensuring “freezeout” of curvature perturbations at super horizon scales, one can get a tensor-scalar ratio $r \sim \mathcal{O}(10^{-3})$ in the context of slow-roll scenarios. In section 5, we summarize our results and give some arguments to show the possibility of identifying the inflaton, responsible for slow-roll inflation, to also be a dark matter candidate as well as a quintessence field for sub-Planckian axions.

2 Review of Large Volume Slow-Roll Axionic Inflationary Scenarios Including Non-Perturbation α' Corrections

Let us first summarize the results of our previous works (section 4 of [1]) and [2]. With the inclusion of perturbative (using [42]) and non-perturbative (using [43]) α' -corrections as well as the loop corrections (using [44]), the Kähler potential for the two-parameter “Swiss-Cheese” Calabi-Yau expressed as a projective variety in $\mathbf{WCP}^4[1, 1, 1, 6, 9]$, can be shown to be given by:

$$\begin{aligned}
K = & -\ln(-i(\tau - \bar{\tau})) - \ln\left(-i \int_{CY_3} \Omega \wedge \bar{\Omega}\right) \\
& -2 \ln\left[\mathcal{V} + \frac{\chi(CY_3)}{2} \sum_{m,n \in \mathbf{Z}^2/(0,0)} \frac{(\bar{\tau} - \tau)^{\frac{3}{2}}}{(2i)^{\frac{3}{2}} |m + n\tau|^3}\right. \\
& \left. -4 \sum_{\beta \in H_2^-(CY_3, \mathbf{Z})} n_\beta^0 \sum_{m,n \in \mathbf{Z}^2/(0,0)} \frac{(\bar{\tau} - \tau)^{\frac{3}{2}}}{(2i)^{\frac{3}{2}} |m + n\tau|^3} \cos\left((n + m\tau)k_a \frac{(G^a - \bar{G}^a)}{\tau - \bar{\tau}} - mk_a G^a\right)\right] \\
& + \frac{C_s^{KK(1)}(U_\alpha, \bar{U}_{\bar{\alpha}})\sqrt{\tau_s}}{\mathcal{V}\left(\sum_{(m,n) \in \mathbf{Z}^2/(0,0)} \frac{(\tau - \bar{\tau})}{2i} \frac{1}{|m + n\tau|^2}\right)} + \frac{C_b^{KK(1)}(U_\alpha, \bar{U}_{\bar{\alpha}})\sqrt{\tau_b}}{\mathcal{V}\left(\sum_{(m,n) \in \mathbf{Z}^2/(0,0)} \frac{(\tau - \bar{\tau})}{2i} \frac{1}{|m + n\tau|^2}\right)}. \tag{1}
\end{aligned}$$

In (1), the first line and $-2 \ln(\mathcal{V})$ are the tree-level contributions. The second (excluding the volume factor in the argument of the logarithm) and third lines are the perturbative and non-perturbative α' corrections. $\{n_\beta^0\}$ are the genus-zero Gopakumar-Vafa invariants that count the number of genus-zero rational curves. The fourth line is the 1-loop contribution; τ_s is the volume of the “small” divisor and τ_b is the volume of the “big” divisor. The loop-contributions arise from KK modes corresponding to closed string or 1-loop open-string exchange between $D3$ - and $D7$ -(or $O7$ -planes)branes wrapped around the “s” and “b” divisors. Note that the two divisors for $\mathbf{WCP}^4[1, 1, 1, 6, 9]$, do not intersect (See [45]) implying that there is no contribution from winding modes corresponding to strings winding non-contractible 1-cycles in the intersection locus corresponding to stacks of intersecting $D7$ -branes wrapped around the “s” and “b” divisors. One sees

from (1) that in the LVS limit, loop corrections are sub-dominant as compared to the perturbative and non-perturbative α' corrections.

To summarize the result of section 4 of [1], one gets the following potential:

$$\begin{aligned}
V \sim & \frac{\mathcal{Y}\sqrt{\ln\mathcal{Y}}}{\mathcal{V}^{2n^s+2}} e^{-2\phi} \frac{\left(\sum_{n^s} n^s \sum_{m^a} e^{-\frac{m^2}{2g_s} + \frac{m_a b^a n^s}{g_s} + \frac{n^s \kappa_{1ab} b^a b^b}{2g_s}} \right)^2}{|f(\tau)|^2} \\
& + \sum_{n^s} \frac{W \ln \mathcal{Y}}{\mathcal{V}^{n^s+2}} \left(\frac{\theta_{n^s}(\bar{\tau}, \bar{G})}{f(\eta(\bar{\tau}))} \right) e^{-in^s(-\tilde{\rho}_1 + \frac{1}{2}\kappa_{1ab} \frac{\bar{\tau} G^a - \tau \bar{G}^a}{(\bar{\tau} - \tau)} \frac{(G^b - \bar{G}^b)}{(\bar{\tau} - \tau)} - \frac{1}{2}\kappa_{1ab} \frac{G^a(G^b - \bar{G}^b)}{(\tau - \bar{\tau})})} + c.c. \\
& + \sum_{k^1, k^2} \frac{|W|^2}{\mathcal{V}^3} \left(\frac{3k_2^2 + k_1^2}{k_1^2 - k_2^2} \right) \frac{\left| \sum_c \sum_{n, m \in \mathbf{Z}^2/(0,0)} e^{-\frac{3\phi}{2}} A_{n, m, n_{kc}}(\tau) \sin(nk.b + mk.c) \right|^2}{\sum_{c'} \sum_{m', n' \in \mathbf{Z}^2/(0,0)} e^{-\frac{3\phi}{2}} |n + m\tau|^3 |A_{n', m', n_{kc'}}(\tau)|^2 \cos(n'k.b + m'k.c)} + \frac{\xi |W|^2}{\mathcal{V}^3},
\end{aligned} \tag{2}$$

where \mathcal{V} is the overall volume of the Swiss-Cheese Calabi-Yau, n^s is the $D3$ -brane instanton quantum number, m^a 's are the $D1$ instanton numbers and $f(\tau)$ is an appropriate modular function. The expressions for \mathcal{Y} , the holomorphic Jacobi theta function $\theta_{n^\alpha}(\tau, G)$ and $A_{n, m, n_{kc}}(\tau)$ are defined as:

$$\begin{aligned}
\mathcal{Y} \equiv & \mathcal{V}_E + \frac{\chi}{2} \sum_{m, n \in \mathbf{Z}^2/(0,0)} \frac{(\tau - \bar{\tau})^{\frac{3}{2}}}{(2i)^{\frac{3}{2}} |m + n\tau|^3} \\
& - 4 \sum_{\beta \in H_2^-(CY_3, \mathbf{Z})} n_\beta^0 \sum_{m, n \in \mathbf{Z}^2/(0,0)} \frac{(\tau - \bar{\tau})^{\frac{3}{2}}}{(2i)^{\frac{3}{2}} |m + n\tau|^3} \cos \left((n + m\tau) k_a \frac{(G^a - \bar{G}^a)}{\tau - \bar{\tau}} - m k_a G^a \right), \\
\theta_{n^\alpha}(\tau, G) = & \sum_{m_a} e^{\frac{i\tau m^2}{2}} e^{in^\alpha G^a m_a}, \\
A_{n, m, n_{kc}}(\tau) \equiv & \frac{(n + m\tau) n_{kc}}{|n + m\tau|^3}.
\end{aligned} \tag{3}$$

Also, G^a are defined by $G^a \equiv c^a - \tau b^a$ (where c^a 's and b^a 's are defined through the real RR two-form potential $C_2 = c_a \omega^a$ and the real NS-NS two-form potential $B_2 = b_a \omega^a$).

On comparing (2) with the analysis of [46], one sees that for generic values of the moduli $\rho_\alpha, G^a, k^{1,2}$ and $\mathcal{O}(1)$ $W_{c.s.}$, and n^s (the $D3$ -brane instanton quantum number)=1, analogous to [46], the second term dominates; the third term is a new term. However, as in KKLT scenarios (See [47]), $W_{c.s.} \ll 1$; we would henceforth assume that the fluxes and complex structure moduli have been so fine tuned/fixed that $W \sim W_{n.p.}$. We assume that the fundamental-domain-valued b^a 's satisfy: $\frac{|b^a|}{\pi} < 1^3$. This implies that for $n^s > 1$, the first term in (2) - $|\partial_{\rho^s} W_{np}|^2$ - a positive definite term, is the most dominant. In the same, ρ^s is the volume of the small divisor complexified by RR 4-form axions. Hence, if a minimum exists, it will be positive. As shown in [1], the potential can be extremized along the locus:

$$mk.c + nk.b = N_{(m, n; k^a)} \pi \tag{4}$$

³If one puts in appropriate powers of the Planck mass M_p , $\frac{|b^a|}{\pi} < 1$ is equivalent to $|b^a| < \pi M_p$, i.e., NS-NS axions are sub-Planckian in units of πM_p .

with $n^s > 1$ and for all values of the $D1$ -instanton quantum numbers m^a .⁴

As shown in section 3 of [2], it turns out that the locus $nk.b + mk.c = N\pi$ for $|b^a| < \pi$ and $|c^a| < \pi$ corresponds to a flat saddle point with the NS-NS axions providing a flat direction. For all directions in the moduli space with $W_{c.s.} \sim \mathcal{O}(1)$ and away from $D_i W_{cs} = D_\tau W = 0 = \partial_{c^a} V = \partial_{b^a} V = 0$, the $\mathcal{O}(\frac{1}{\mathcal{V}^2})$ contribution of $\sum_{\alpha, \bar{\beta} \in c.s.} (G^{-1})^{\alpha\bar{\beta}} D_\alpha W_{cs} \bar{D}_{\bar{\beta}} \bar{W}_{cs}$ dominates over (2), ensuring that there must exist a minimum, and given the positive definiteness of the potential, this will be a dS minimum. There has been no need to add any $\overline{D3}$ -branes as in KKLT to generate a dS vacuum.

3 Finite f_{NL}

We now proceed to showing the possibility of getting finite values for the non-linearity parameter f_{NL} in two different contexts. First, we show the same for slow-roll inflationary scenarios. Second, we show the same when the slow-roll conditions are violated.

3.1 Slow-Roll Inflationary Scenarios

In [2], we discussed the possibility of getting slow roll inflation along a flat direction provided by the NS-NS axions starting from a saddle point and proceeding to the nearest dS minimum. In what follows, we will assume that the volume moduli for the small and big divisors and the axion-dilaton modulus have been stabilized. All calculations henceforth will be in the axionic sector - ∂_a will imply ∂_{G^a} in the following. On evaluation of the slow-roll inflation parameters (in $M_p = 1$ units) $\epsilon \equiv \frac{G^{ij} \partial_i V \partial_j V}{2V^2}$, $\eta \equiv$ the most negative eigenvalue of the matrix $N^i_j \equiv \frac{G^{ik} (\partial_k \partial_j V - \Gamma_{jk}^l \partial_l V)}{V}$ with Γ_{jk}^l being the affine connection components, we found that $\epsilon \sim \frac{(n^s)^2}{(k^2 g_s^{\frac{3}{2}} \Delta) \mathcal{V}}$ and $\eta \sim \frac{1}{k^2 g_s^{\frac{3}{2}} \Delta} [g_s n^s \kappa_{1ab} + \frac{(n^s)^2}{\sqrt{\mathcal{V}}} \pm n^s k^2 g_s^{\frac{3}{2}} \Delta]^5$ where $\Delta \equiv \frac{\sum_{\beta \in H_2^-(CY_3, \mathbf{Z})} n_\beta^0}{\mathcal{V}}$ and we have chosen Calabi-Yau volume \mathcal{V} to be such that $\mathcal{V} \sim e^{\frac{4\pi^2}{g_s}}$ (similar to [48]). Using Castelnuovo's theory of study of moduli spaces that are fibrations of Jacobian of curves over the moduli space of their deformations, for compact Calabi-Yau's expressed as projective varieties in weighted complex projective spaces (See [49]) one sees that for appropriate degrees of the holomorphic curve, the genus-0 Gopakumar-Vafa invariants can be very large to compensate the volume factor appearing in the expression for η . Hence the slow-roll conditions can be satisfied, and in particular, there is no “ η ”-problem. By investigating the eigenvalues of the Hessian, we showed (in [2]) that one could identify a linear combination of the NS-NS axions (“ $k_2 b^2 + k_1 b^1$ ”) with the inflaton and the slow-roll inflation starts from the aforementioned saddle-point and ends when the slow-roll conditions were violated, which most probably corresponded to the nearest dS minimum, one can show that (in $M_p = 1$ units)

$$N_e = - \int_{\text{in: Saddle Point}}^{\text{fin: dS Minimum}} \frac{1}{\sqrt{\epsilon}} d\mathcal{I} \sim \frac{k g_s^{3/4} \sqrt{\sum_{\beta \in H_2} n_\beta^0}}{n^s}. \quad (5)$$

We will see that one can get $N_e \sim 60$ e-foldings in the context of slow roll as well as slow roll violating scenarios. Now before explaining how to get the non-linear parameter “ f_{NL} ” relevant to studies of non-Gaussianities, to be $\mathcal{O}(10^{-2})$ in our slow-roll LVS Swiss-Cheese orientifold setup, let us summarize the

⁴Considering the effect of axionic shift symmetry (of the b^a axions) on the $D1$ -instanton superpotential ($W_{D1\text{-instanton}}$), one can see that m^a is valued in a lattice with coefficients being integral multiple of 2π .

⁵The g_s and k -dependence of ϵ and η was missed in [2]. The point is that the extremization of the potential w.r.t. b^a 's and c^a 's in the large volume limit yields a saddle point at $\sin(nk.b + mk.c) = 0$ and those maximum degree- k^a holomorphic curves β for which $b^a \sim -m^a/\kappa$ (assuming that $\frac{nk.m}{\pi\kappa} \in \mathbf{Z}$).

formalism and results of [3] in which the authors analyze the primordial non-Gaussianity in multi-scalar field inflation models (without the explicit form of the potential) using the slow-roll conditions and the δN formalism of ([31]) as the basic inputs.

Assuming that the time derivative of scalar field $\phi^a(t)$ is not independent of $\phi^a(t)$ (as in the case of standard slow-roll inflation) the background e -folding number between an initial hypersurface at $t = t_*$ and a final hypersurface at $t = t_c$ (which is defined by $N \equiv \int H dt$) can be regarded as a function of the homogeneous background field configurations $\phi^a(t_*)$ and $\phi^a(t_c)$ (on the initial and final hypersurface at $t = t_*$ and $t = t_c$ respectively). i.e.

$$N \equiv N(\phi^a(t_c), \phi^a(t_*)) . \quad (6)$$

By considering t_c to be a time when the background trajectories have converged, the curvature perturbation ζ evaluated at $t = t_c$ is given by $\delta N(t_c, \phi^a(t_*))$ (using the δN formalism). After writing the $\delta N(t_c, \phi^a(t_*))$ upto second order in field perturbations $\delta\phi^a(t_*)$ (on the initial flat hypersurface at $t = t_*$) the curvature perturbation $\zeta(t_c)$ becomes

$$\zeta(t_c) \simeq \delta N(t_c, \phi_*) = \partial_a N^* \delta\phi_*^a + \frac{1}{2} \partial_a \partial_b N^* \delta\phi_*^a \delta\phi_*^b , \quad (7)$$

and using the power spectrum correlator equations and $\zeta(\mathbf{x}) = \zeta_G(\mathbf{x}) - \frac{3}{5} f_{NL} \zeta_G^2(\mathbf{x})$, where $\zeta_G(\mathbf{x})$ represents the Gaussian part, one can arrive at

$$-\frac{6}{5} f_{NL} \simeq \frac{\partial^a N_* \partial^b N_* \partial_a \partial_b N^*}{(\partial_c N^* \partial^c N_*)^2} \quad (8)$$

with the assumption that the field perturbation on the initial flat hypersurface, $\delta\phi_*^a$, is Gaussian.

For the generalization of the above in the context of non-Gaussianities, the authors assumed the so called “relaxed” slow-roll conditions (RSRC) (which is $\epsilon \ll 1$ and $|\eta_{ab}| \ll 1$) for all the scalar fields, and introduce a time t_f , at which the RSRC are still satisfied. Then for calculating $\zeta(t_c)$, they express $\delta\phi^a(t_f)$ in terms of $\delta\phi_*^a$, with the scalar field expanded as $\phi^a \equiv \phi_0^a + \delta\phi^a$ and then evaluate $N(t_c, \phi^a(t_f))$ (the e -folding number to reach $\phi^{a(0)}(t_c)$ starting with $\phi^a = \phi^a(t_f)$) and with the calculation of $\zeta(t_c)$ in terms of derivatives of field variations of N^f (making use of the background field equations in variable N instead of time variable) and comparing the same with (7) and using (8) one arrives at the following general expression for the non-linear parameter f_{NL} :

$$-\frac{6}{5} f_{NL} = \frac{\partial_a \partial_b N^f \Lambda_{a'}^f \mathcal{G}^{a'a''} \partial_{a''} N_* \Lambda_{b'}^b \mathcal{G}^{b'b''} \partial_{b''} N_* + \int_{N_*}^{N_f} dN \partial_c N Q_{df}^c \Lambda_{d'}^d \mathcal{G}^{d'd''} \partial_{d''} N_* \Lambda_{f'}^f \mathcal{G}^{f'f''} \partial_{f''} N_*}{(\mathcal{G}^{kl} \partial_k N^* \partial_l N_*)^2}, \quad (9)$$

with the following two constraints (See [3]) required:

$$\left| \frac{\mathcal{G}^{ab} (\partial_b V)_{;a}}{V} \right| \ll \sqrt{\frac{\mathcal{G}^{ab} \partial_a V \partial_b V}{V^2}} \text{ and } \left| Q_{bc}^a \right| \ll \sqrt{\frac{\mathcal{G}^{ab} \partial_a V \partial_b V}{V^2}}, \quad (10)$$

the semicolon implying a covariant derivative involving the affine connection. In (9) and (10), \mathcal{G}^{ab} 's are the components of the moduli space metric along the axionic directions given as,

$$\mathcal{G}^{ab} \sim \frac{\mathcal{Y}}{k^2 g_s^{\frac{3}{2}} \sum_{\beta \in H_2^-(CY_3, \mathbf{Z})} n_\beta^0} \equiv \frac{1}{k^2 g_s^{\frac{3}{2}} \Delta} \quad (11)$$

Further,

$$\begin{aligned}\Lambda_b^a &\equiv \left(T e^{\int_{N_*}^{N_f} P(N) dN} \right)_b^a, \quad P_b^a \equiv \left[-\frac{\partial_{a'}(\mathcal{G}^{aa'} \partial_b V)}{V} + \frac{\mathcal{G}^{aa'} \partial_{a'} V \partial_b V}{V^2} \right]; \\ Q_{bc}^a &\equiv \left[-\frac{\partial_{a'}(\mathcal{G}^{aa'} \partial_b \partial_c V)}{V} + \frac{\partial_{a'}(\mathcal{G}^{aa'} \partial_b V) \partial_c V}{V^2} + \frac{\mathcal{G}^{aa'} \partial_{a'} V \partial_b \partial_c V}{V^2} - 2 \frac{\mathcal{G}^{aa'} \partial_{a'} V \partial_b V \partial_c V}{V^3} \right],\end{aligned}\quad (12)$$

where V is the scalar potential and as the number of e-folding is taken as a measure of the period of inflation (and hence as the time variable), the expression for Λ_J^I above, has a time ordering T with the initial and final values of number of e-foldings N_* and N_f respectively. From the definition of P_J^I and Λ_J^I , one sees that during the slow-roll epoch, $\Lambda_J^I = \delta_J^I$.

After using (2) along with:

$$\begin{aligned}\sum_{m_a \in 2\mathbf{Z}\pi} e^{-\frac{m^2}{2g_s} + \frac{m_a b^a n^s}{g_s} + \frac{n^s \kappa_{1ab} b^a b^b}{2g_s}} &\sim 1 \\ \sum_{m_a \in 2\mathbf{Z}\pi} m_a e^{-\frac{m^2}{2g_s} + \frac{m_a b^a n^s}{g_s} + \frac{n^s \kappa_{1ab} b^a b^b}{2g_s}} &\sim e^{-\frac{2\pi^2}{g_s}} \sim \frac{1}{\sqrt{\mathcal{V}}},\end{aligned}\quad (13)$$

for sub-planckian b^a 's, one arrives at the following results (along the slow-roll direction $\sin(nk_a b^a + mk_a c^a) = 0$) :

$$\begin{aligned}\frac{\partial_a V}{V} &\sim \frac{n^s}{\sqrt{\mathcal{V}}}; \quad \frac{\partial_a \partial_b V}{V} \sim g_s n^s \kappa_{1ab} + \frac{(n^s)^2}{\sqrt{\mathcal{V}}} \pm n^s k^2 g_s^{\frac{7}{2}} \Delta, \\ \frac{\partial_a \partial_b \partial_c V}{V} &\sim \frac{n^s}{\sqrt{\mathcal{V}}} \left[g_s (n^s) \kappa_{1ab} + (n^s)^2 \pm g_s^{\frac{7}{2}} n^s k^2 \Delta \right]\end{aligned}\quad (14)$$

Further using the above, one sees that the ϵ and η parameters along with Q_{bc}^a (appearing in the expression of f_{NL}) are given as under:

$$\epsilon \sim \frac{(n^s)^2}{\mathcal{V} g_s^{\frac{3}{2}} k^2 \Delta}; \quad \eta \sim \frac{1}{g_s^{\frac{3}{2}} k^2 \Delta} \left[g_s n^s \kappa_{1ab} + \frac{(n^s)^2}{\sqrt{\mathcal{V}}} \pm n^s k^2 g_s^{\frac{7}{2}} \Delta \right], \quad (15)$$

and

$$Q_{bc}^a \sim \frac{n^s}{g_s^{\frac{3}{2}} k^2 \Delta \sqrt{\mathcal{V}}} \left[(n^s)^2 + \frac{(n^s)^2}{\sqrt{\mathcal{V}}} - \frac{(n^s)^2}{\mathcal{V}} \pm n^s k^2 g_s^{\frac{7}{2}} \Delta \right] \quad (16)$$

Now in order to use the expression for f_{NL} , the first one of required constraints (10) results in the following inequality:

$$|\delta| \equiv \left| g_s n^s \kappa_{1ab} + \frac{(n^s)^2}{\sqrt{\mathcal{V}}} - n^s g_s^{\frac{7}{2}} k^2 \Delta \right| \ll n^s \sqrt{\frac{g_s^{\frac{3}{2}} k^2 \Delta}{\mathcal{V}}} \quad (17)$$

Now we solve the above inequality for say $|\delta| \sim \frac{1}{\mathcal{V}}$, which is consistent with the constraint requirement along with the following relation

$$\Delta \equiv \frac{\sum_{\beta \in H_2^-(CY_3, \mathbf{Z})} n_\beta^0}{\mathcal{Y}} \sim \frac{1}{k^2 g_s^{\frac{7}{2}}} \left[g_s \kappa_{1ab} + \frac{n^s}{\sqrt{\mathcal{V}}} \right] \sim \frac{1}{k^2 g_s^{\frac{5}{2}}} \quad (18)$$

The second constraint is

$$\left| (n^s)^2 + \frac{(n^s)^2}{\sqrt{\mathcal{V}}} + \frac{(n^s)^2}{\mathcal{V}} - g_s^{\frac{7}{2}} k^2 (n^s) \Delta \right| \ll \sqrt{k^2 \Delta g_s^{\frac{3}{2}}} \quad (19)$$

Given that we are not bothering about precise numerical factors, we will be happy with “<” instead of a strict “ \ll ” in (19). Using (18) in the previous expressions (15,16) for ϵ, η and Q_{bc}^a , we arrive at the final expression for the slow-roll parameters ϵ, η and Q_{bc}^a as following:

$$\begin{aligned} \epsilon &\sim G^{ab} \frac{(n^s)^2}{\mathcal{V}} \sim \frac{g_s (n^s)^2}{\mathcal{V}}; |\eta| \sim G^{ab} |\delta| \sim \frac{g_s}{\mathcal{V}} \\ (Q_{bc}^a)_{max} &\sim G^{ab} \sqrt{k^2 \Delta g_s^{\frac{3}{2}}} \left(\frac{n^s}{\sqrt{\mathcal{V}}} \right) \sim n^s \sqrt{\frac{g_s}{\mathcal{V}}} \end{aligned} \quad (20)$$

As the number of e-foldings satisfies $\partial_I N = \frac{V}{\partial_I V} \sim \frac{\sqrt{\mathcal{V}}}{n^s \sqrt{g_s}}$, which is almost constant and hence $\partial_I \partial_J N \sim 0$. Consequently the first term of (9) is negligible and the maximum contribution to the non-Gaussianities parameter f_{NL} coming from the second term is given by:

$$\frac{\int_{N_*}^{N_f} dN \partial_c N Q_{bc}^a \Lambda_{b'}^b \mathcal{G}^{b'b''} \partial_{b''} N \Lambda_{c'}^c \mathcal{G}^{c'c''} \partial_{c''} N}{(\mathcal{G}^{df} \partial_d N \partial_f N)^2} \leq (Q_{bc}^a)_{max} \sim n^s \sqrt{\frac{g_s}{\mathcal{V}}}. \quad (21)$$

This way, for Calabi-Yau volume $\mathcal{V} \sim 10^6$, D3-instanton number $n^s = \mathcal{O}(10)$ with $n^s \sim g_s \sim k^2$ implying the slow-roll parameters⁶ $\epsilon \sim 0.00028, |\eta| \sim 10^{-6}$ with the number of e-foldings $N_e \sim 60$, one obtains the maximum value of the non-Gaussianities parameter $(f_{NL})_{max} \sim 10^{-2}$. Further if we choose the stabilized Calabi-Yau volume $\mathcal{V} \sim 10^5$ with $n^s = \mathcal{O}(10)$, we find $\epsilon \sim 0.0034, |\eta| \sim 10^{-4}$ with the number of e-foldings $N_e \sim 17$ and the maximum possible $(f_{NL})_{max} \sim 3 \times 10^{-2}$. The above mentioned values of ϵ and η parameters can be easily realized in our setup with the appropriate choice of holomorphic isometric involution as part of the Swiss-Cheese orientifold. This way we have realized $\mathcal{O}(10^{-2})$ non-gaussianities parameter f_{NL} in slow-roll scenarios of our LVS Swiss-Cheese orientifold setup.

3.2 Slow-Roll Conditions are Violated

We will now show that it is possible to obtain $\mathcal{O}(1)$ f_{NL} while looking for non-Gaussianities in curvature perturbations when the slow-roll conditions are violated. We will follow the formalism developed in [4] to discuss evaluation of f_{NL} in scenarios wherein the slow-roll conditions are violated. Before that let us summarize the results of [4] in which the authors analyze the non-Gaussianity of the primordial curvature perturbation generated on super-horizon scales in multi-scalar field inflation models *without imposing the slow-roll conditions* and using the δN formalism of ([31]) as the basic input.

Consider a model with n -component scalar field ϕ^a . Now consider the perturbations of the scalar fields in constant N gauge as

$$\delta \phi^A(N) \equiv \phi^A(\lambda + \delta \lambda; N) - \phi^A(\lambda; N), \quad (22)$$

where the short-hand notation of [4] is used - $X^A \equiv X_i^a (i = 1, 2) = (X_1^a \equiv X^a, X_2^a \equiv \frac{dX^a}{dN})^7$, and where λ^A 's are the $2n$ integral constants of the background field equations. After using the decomposition of the fields ϕ^A

⁶These values are allowed for the curvature perturbation freeze-out at the superhorizon scales, which is discussed in the section pertaining to finite tensor-to scalar ratio.

⁷We have modified the notations of [4] a little.

up to second order in δ (defined through $\delta\tilde{\phi}^A = \delta\tilde{\phi}_{(1)}^A + \frac{1}{2}\delta\tilde{\phi}_{(2)}^A$; to preserve covariance under general coordinate transformation in the moduli space, the authors of [4] define: $(\delta\tilde{\phi}_{(1)})_1^a \equiv \frac{d\phi^a}{d\lambda}\delta\lambda$, $(\delta\tilde{\phi}_{(2)})_1^a \equiv \frac{D}{d\lambda}\frac{d\phi^a}{d\lambda}(\delta\lambda)^2$ and $(\delta\tilde{\phi}_{(1)})_2^a \equiv \frac{D\phi_2^a}{d\lambda}\delta\lambda$, $(\delta\tilde{\phi}_{(2)})_2^a \equiv \frac{D^2\phi_2^a}{d\lambda^2}(\delta\lambda)^2$), one can solve the evolution equations for $\delta\tilde{\phi}_{(1)}^A$ and $\delta\tilde{\phi}_{(2)}^A$. The equation for $\delta\tilde{\phi}_{(2)}^A$ is simplified with the choice of integral constants such that $\lambda^A = \phi^A(N_*)$ implying $\delta\tilde{\phi}^A(N_*) = \delta\lambda^A$ and hence $\delta\tilde{\phi}_{(2)}^A(N)$ vanishing at N_* . Assuming N_* to be a certain time soon after the relevant length scale crossed the horizon scale (H^{-1}), during the scalar dominant phase and N_c to be a certain time after the complete convergence of the background trajectories has occurred and using the so called δN formalism one gets

$$\zeta \simeq \delta N = \tilde{N}_A \delta\tilde{\phi}^A + \frac{1}{2} \tilde{N}_{AB} \delta\tilde{\phi}^A \delta\tilde{\phi}^B + \dots \quad (23)$$

Now taking N_f to be certain late time during the scalar dominant phase and using the solutions for $\delta\phi_{(1)}^A$ and $\delta\phi_{(2)}^A$ for the period $N_* < N < N_f$, one obtains the expressions for \tilde{N}_{A*} and \tilde{N}_{AB*} (to be defined below) and finally writing the variance of $\delta\tilde{\phi}_*^A$ (defined through $\langle \delta\tilde{\phi}_*^A \delta\tilde{\phi}_*^B \rangle \simeq A^{AB} \left(\frac{H_*}{2\pi}\right)^2$ including corrections to the slow-roll terms in A^{ab} based on [20, 21]), and using the basic definition of the non-linear parameter f_{NL} as the the magnitude of the bispectrum of the curvature perturbation ζ , one arrives at a general expression for f_{NL} (for non slow roll cases)[4]. For our present interest, the expression for f_{NL} for the non-slow roll case is given by

$$-\frac{6}{5}f_{NL} = \frac{\tilde{N}_{AB}^f \Lambda_{A'}^A(N_f, N_*) A^{A'A''} \tilde{N}_{A''}^* \Lambda_{B'}^B A^{B'B''} \tilde{N}_{B''}^* + \int_{N_*}^{N_f} dN \tilde{N}_C \tilde{Q}_{D\mathcal{F}}^C \Lambda_{D'}^D A^{D'D''} \tilde{N}_{D''}^* \Lambda_{F'}^F A^{F'F''} \tilde{N}_{F''}^*}{(A^{\mathcal{KL}} \tilde{N}_{\mathcal{K}}^* \tilde{N}_{\mathcal{L}}^*)^2}, \quad (24)$$

where again the index \mathcal{A} represents a pair of indices $_i^a$, $i = 1$ corresponding to the field b^a and $i = 2$ corresponding to $\frac{db^a}{dN}$. Further,

$$\begin{aligned} A_{11}^{ab} &= \mathcal{G}^{ab} + \left(\sum_{m_1, m_2, m_3, m_4, m_5}^{<\infty} \left(\left\| \frac{d\phi^a}{dN} \right\|^2 \right)^{m_1} \left(\frac{1}{H} \frac{dH}{dN} \right)^{m_2} \epsilon^{m_3} \eta^{m_4} \right)^{ab}, \\ A_{12}^{ab} &= A_{21}^{ab} = \frac{\mathcal{G}^{aa'} \partial_{a'} V \mathcal{G}^{bb'} \partial_{b'} V}{V^2} - \frac{\partial_{a'} (\mathcal{G}^{aa'} \mathcal{G}^{bb'} \partial_{b'} V)}{V}, \\ A_{22}^{ab} &= \left(\frac{\mathcal{G}^{aa'} \partial_{a'} V \partial_c V}{V^2} - \frac{\partial_{a'} (\mathcal{G}^{aa'} \partial_c V)}{V} \right) \left(\frac{\mathcal{G}^{cc'} \partial_{c'} V \mathcal{G}^{bb'} \partial_{b'} V}{V^2} - \frac{\partial_{c'} (\mathcal{G}^{cc'} \mathcal{G}^{bb'} \partial_{b'} V)}{V} \right), \end{aligned} \quad (25)$$

where in A_{11}^{ab} , based on [20, 21], assuming the non-Gaussianity to be expressible as a finite-degree polynomial in higher order slow-roll parameter corrections. In (24), one defines:

$$\Lambda_B^A = \left(T e^{\int_{N_*}^{N_f} dN \tilde{P}(N)} \right)_B^A; \quad (26)$$

$\tilde{N}_A, \tilde{N}_{AB}, \tilde{P}_B^A$ and \tilde{Q}_{BC}^A ⁸ will be defined momentarily. The equations of motion

$$\begin{aligned} \frac{d^2 b^a}{dN^2} + \Gamma_{bc}^a \frac{db^b}{dN} \frac{db^c}{dN} + \left(3 + \frac{1}{H} \frac{dH}{dN} \right) \frac{db^a}{dN} + \frac{\mathcal{G}^{ab} \partial_b V}{H^2} &= 0, \\ H^2 &= \frac{1}{3} \left(\frac{1}{2} H^2 \left\| \frac{db^a}{dN} \right\|^2 + V \right) \end{aligned} \quad (27)$$

⁸We have modified the notations of [4] for purposes of simplification.

yield

$$\frac{1}{H} \frac{dH}{dN} = \frac{6 \left(\frac{2V}{H^2} - 6 \right) - 2\Gamma_{bc}^a \frac{db_a}{dN} \frac{db^b}{dN} \frac{db^c}{dN}}{12}. \quad (28)$$

For slow-roll inflation, $H^2 \sim \frac{V}{3}$; the Friedmann equation in (27) implies that $H^2 > \frac{V}{3}$ when slow-roll conditions are violated. The number of e-foldings away from slow-roll is given by: $N \sim \int \frac{db^a}{\|\frac{db^b}{dN}\|}$, which using the Friedmann equation implies $N \sim \int \frac{db^a}{\sqrt{1 - \frac{V}{3H^2}}}$.

We would require $\epsilon \ll 1$ and $|\eta| \mathcal{O}(1)$ to correspond to a slow-roll violating scenario. Now, writing the potential $V = V_0 \frac{\sqrt{\ln \mathcal{Y}}}{\mathcal{Y}^{2n^s+1}} \left(\sum_{m \in 2\mathbf{Z}\pi} e^{-\frac{m^2}{2g_s} + \frac{n^s}{g_s} m \cdot b + \frac{\kappa_{1ab} b^a b^b}{2g_s}} \right)^2$, $\partial_{\mathcal{G}^a} V = 0$ implies:

$$\begin{aligned} \partial_{\mathcal{G}^a} V \sim & -\frac{g_s \sqrt{\ln \mathcal{Y}} n^s}{\mathcal{Y}^{2n^s+1}} \sum_{\beta \in H_2^-(CY_3, \mathbf{Z})} \frac{n_\beta^0}{\mathcal{Y}} \sin(nk \cdot b + mk \cdot c) k^a g_s^{\frac{3}{2}} \left(\sum_{m \in 2\mathbf{Z}\pi} e^{-\frac{m^2}{2g_s} + \frac{n^s}{g_s} m \cdot b + \frac{\kappa_{1ab} b^a b^b}{2g_s}} \right)^2 \\ & + \frac{g_s \sqrt{\ln \mathcal{Y}}}{\mathcal{Y}^{2n^s+1}} \left(\sum_{m \in 2\mathbf{Z}\pi} e^{-\frac{m^2}{2g_s} + \frac{n^s}{g_s} m \cdot b + \frac{\kappa_{1ab} b^a b^b}{2g_s}} \right) \frac{n^s}{g_s} \sum_{m \in 2\mathbf{Z}\pi} (m^a + \kappa_{1ab} b^b) \left(\sum_{m^a \in 2\mathbf{Z}\pi} e^{-\frac{m^2}{2g_s} + \frac{n^s}{g_s} m \cdot b + \frac{\kappa_{1ab} b^a b^b}{2g_s}} \right) \end{aligned} \quad (29)$$

which using $\sum_{m \in 2\mathbf{Z}\pi} e^{-\frac{m^2}{2g_s} + \frac{n^s}{g_s} m \cdot b + \frac{\kappa_{1ab} b^a b^b}{2g_s}} \sim e^{\frac{\kappa_{1ab} b^a b^b}{2g_s}}$ and $\sum_{m \in 2\mathbf{Z}\pi} (m^a + \kappa_{1ab} b^b) \left(\sum_{m^a \in 2\mathbf{Z}\pi} e^{-\frac{m^2}{2g_s} + \frac{n^s}{g_s} m \cdot b + \frac{\kappa_{1ab} b^a b^b}{2g_s}} \right) \sim \kappa_{1ab} b^b e^{\frac{\kappa_{1ab} b^a b^b}{2g_s}}$, one obtains:

$$\sum_{\beta \in H_2^-(CY_3, \mathbf{Z})} \frac{n_\beta^0}{\mathcal{Y}} \sin(nk \cdot b + mk \cdot c) k^a \sim \frac{b^a}{g_s^{\frac{5}{2}}}. \quad (30)$$

Slow-roll scenarios assumed that the LHS and RHS of (30) vanished individually - the same will not be true for slow-roll violating scenarios. Near (30), one can argue that:

$$\begin{aligned} \mathcal{G}^{\mathcal{G}^a \bar{\mathcal{G}}^b} \sim & \frac{b^a b^b - \sqrt{g_s^7 (k^a k^b)^2 \left(\frac{n_\beta^0}{\mathcal{V}} \right)^2 - b^2 g_s^2 k^2}}{b^2 \sqrt{g_s^7 (k^a k^b)^2 \left(\frac{n_\beta^0}{\mathcal{V}} \right)^2 - b^2 g_s^2 k^2} + g_s^7 (k^a k^b)^2 \left(\frac{n_\beta^0}{\mathcal{V}} \right)^2 - b^2 g_s^2 k^2}; \\ \Gamma_{\mathcal{G}^b \mathcal{G}^c}^{\mathcal{G}^a} \sim & \frac{\left(\sqrt{g_s^7 (k^a k^b)^2 \left(\frac{n_\beta^0}{\mathcal{V}} \right)^2 - b^2 g_s^2 k^2} + g_s^2 + \frac{b^2 g_s^2}{\sqrt{g_s^7 (k^a k^b)^2 \left(\frac{n_\beta^0}{\mathcal{V}} \right)^2 - b^2 g_s^2 k^2}} \right)}{b^2 \sqrt{g_s^7 (k^a k^b)^2 \left(\frac{n_\beta^0}{\mathcal{V}} \right)^2 - b^2 g_s^2 k^2} + g_s^7 (k^a k^b)^2 \left(\frac{n_\beta^0}{\mathcal{V}} \right)^2 - b^2 g_s^2 k^2}. \end{aligned} \quad (31)$$

Note, we no longer restrict ourselves to sub-Planckian axions - we only require $|b^a| < \pi$. For $g_s^7 (k^a k^b)^2 \left(\frac{n_\beta^0}{\mathcal{V}} \right)^2 - b^2 g_s^2 k^2 \sim \mathcal{O}(1)$, and the holomorphic isometric involution, part of the Swiss-Cheese orientifolding, assumed to be such that the maximum degree of the holomorphic curve being summed over in the non-perturbative

α' -corrections involving the genus-zero Gopakumar-Vafa invariants are such that $\sum_{\beta} \frac{n_{\beta}^0}{V} \leq \frac{1}{60}, k \sim 3$, we see that (30) is satisfied and

$$\begin{aligned}\mathcal{G}^{\mathcal{G}^a \bar{\mathcal{G}}^b} &\sim \frac{b^2 + \mathcal{O}(1)}{b^2 + \mathcal{O}(1)} \sim \mathcal{O}(1); \\ \Gamma_{\mathcal{G}^b \mathcal{G}^c}^{\mathcal{G}^a} &\sim \frac{b^a (g_s^2 + b^2 g_s^2)}{b^2 + \mathcal{O}(1)} \sim b^a g_s^2.\end{aligned}\tag{32}$$

Hence, the affine connection components, for $b^a \sim \mathcal{O}(1)$, is of $\mathcal{O}(10)$; the curvature components R^a_{bcd} will hence also be finite. Assuming $H^2 \sim V$, the definitions of ϵ and η continue to remain the same as those for slow-roll scenarios and one hence obtains:

$$\begin{aligned}\epsilon &\sim \frac{(n^s)^2 e^{\frac{4\pi n^s b}{g_s} + \frac{b^2}{g_s}} (\pi + b)^2}{V} \sim 10^{-3}, \\ \eta &\sim n^s (1 + n^s b^2) - \frac{b g_s^2 n^s e^{\frac{4\pi n^s b}{g_s} + \frac{b^2}{g_s}} (\pi + b)}{\sqrt{V}} \sim n^s (1 + n^s b) \sim \mathcal{O}(1).\end{aligned}\tag{33}$$

Finally, $\frac{db^a}{dN} \sim \sqrt{1 - \frac{V}{3H^2}} \sim \mathcal{O}(1)$.

We now write out the various components of $\tilde{P}_{\mathcal{B}}^A$, relevant to evaluation of $\Lambda_{\mathcal{B}}^A$ in (26):

$$\begin{aligned}\tilde{P}_{1b}^{a1} &= 0, \\ \tilde{P}_{1b}^{a2} &= \delta_b^a, \\ \tilde{P}_{2b}^{a1} &= -\frac{V}{H^2} \left(\frac{\partial_a (\mathcal{G}^{ac} \partial_c V)}{V} - \frac{\mathcal{G}^{ac} \partial_c V \partial_b V}{V^2} \right) - R^a_{bcd} \frac{db^c}{dN} \frac{db^d}{dN} \sim \mathcal{O}(1), \\ \tilde{P}_{b2}^{a2} &= \mathcal{G}_{bc} \frac{db^a}{dN} \frac{db^c}{dN} + \frac{\mathcal{G}^{ac} \partial_c V}{V} \mathcal{G}_{bf} \frac{db^f}{dN} - \frac{V}{H^2} \delta_b^a \sim \mathcal{O}(1).\end{aligned}\tag{34}$$

Similarly,

$$\begin{aligned}(a) \quad N_a^1 &\sim \frac{1}{\|\frac{db^a}{dN}\|} \sim \mathcal{O}(1), \\ N_a^2 &= 0; \\ (b) \quad N_{ab}^{11} &= N_{ab}^{12} = N_{ab}^{22} = 0, \\ N_{ab}^{21} &\sim \frac{\mathcal{G}_{bc} \frac{db^c}{dN}}{\|\frac{db^d}{dN}\|^3} \sim \mathcal{O}(1); \\ (c) \quad \tilde{N}_a^1 &\equiv N_a^1 - N_b^2 \Gamma_{ca}^b \frac{db^c}{dN} \sim \frac{1}{\|\frac{db^a}{dN}\|} \sim \mathcal{O}(1), \\ \tilde{N}_a^2 &\equiv N_a^2 = 0; \\ (d) \quad \tilde{N}_{ab}^{11} &\equiv N_{ab}^{11} + N_{ca}^{22} \Gamma_{ml}^c \Gamma_{nb}^l \frac{db^m}{dN} \frac{db^n}{dN} + (N_{ac}^{12} + N_{ac}^{21}) \Gamma_{lb}^c \frac{db^l}{dN} - N_c^2 (\nabla_a \Gamma_{lb}^c) \frac{db^l}{dN} - N_c^2 \Gamma_{al}^c \Gamma_{nb}^l \frac{db^n}{dN} \\ &\sim \frac{\mathcal{G}_{cd} \frac{db^d}{dN}}{\|\frac{db^m}{dN}\|^3} \Gamma_{lb}^c \frac{db^l}{dN} \sim \mathcal{O}(1), \\ \tilde{N}_{ab}^{12} &\equiv N_{ab}^{12} - N_c^2 \Gamma_{ab}^c - N_{cb}^{22} \Gamma_{al}^c \frac{db^l}{dN} = 0,\end{aligned}$$

$$\begin{aligned}
\tilde{N}_{ab}^{21} &\equiv N_{ab}^{21} - N_c^2 \Gamma_{ab}^c - N_{ca}^{22} \Gamma_{bl}^c \frac{db^l}{dN} \sim \frac{\mathcal{G}_{bc} \frac{db^c}{dN}}{\|\frac{db^m}{dN}\|^3} \sim \mathcal{O}(1), \\
\tilde{N}_{ab}^{22} &\equiv N_{ab}^{22} = 0.
\end{aligned} \tag{35}$$

Finally,

$$\begin{aligned}
\tilde{Q}_{1bc}^{a11} &= -R_{bcd}^a \frac{db^d}{dN} \sim \mathcal{O}(1), \\
\tilde{Q}_{1bc}^{a21} &= \tilde{Q}_{1bc}^{a12} = \tilde{Q}_{1bc}^{a22} = 0, \\
\tilde{Q}_{2bc}^{a12} &= \frac{\partial_a(\mathcal{G}^{ad} \partial_d V)}{V} \frac{db^l}{dN} \mathcal{G}_{lc} - 2R_{cbl}^a \frac{db^d}{dN} \sim \mathcal{O}(1), \\
\tilde{Q}_{2bc}^{a22} &= \delta_c^a \mathcal{G}_{bd} \frac{db^d}{dN} + \delta_b^a \mathcal{G}_{cd} \frac{db^d}{dN} + \mathcal{G}_{bc} \left(\frac{db^a}{dN} + \frac{\mathcal{G}^{ad} \partial_a V}{V} \right) \sim \mathcal{O}(1), \\
\tilde{Q}_{2bc}^{a11} &= -\frac{V}{H^2} \left(\frac{\partial_a \partial_b (\mathcal{G}^{ad} \partial_d V)}{V} - \frac{\partial_b (\mathcal{G}^{ad} \partial_d V) \partial_c V}{V^2} \right) - (\nabla_c R_{mbl}^a) \frac{db^m}{dN} \frac{db^l}{dN} \sim \mathcal{O}(1), \\
\tilde{Q}_{2bc}^{a21} &= \left(\frac{\partial_c (\mathcal{G}^{df} \partial_f V)}{V} - \frac{\mathcal{G}^{ad} \partial_d V \partial_c V}{V^2} \right) \mathcal{G}_{bd} \frac{db^d}{dN} - R_{lcb}^a \frac{db^l}{dN} \sim \mathcal{O}(1).
\end{aligned} \tag{36}$$

So, substituting (25), (35)-(36) into (24), one sees that $f_{NL} \sim \mathcal{O}(1)$.

After completion of this work, we were informed about [52] wherein observable values of f_{NL} may be obtained by considering loop corrections.

4 Finite Tensor-Scalar Ratio and Loss of Scale Invariance

We now turn to looking for “finite” values of ratio of amplitudes of tensor and scalar perturbations, “ r ”. Using the Hamilton-Jacobi formalism (See [23] and references therein), which is suited to deal with beyond slow-roll approximations as well, the mode $u_k(y)$ - $y \equiv \frac{k}{aH}$ - corresponding to scalar perturbations, satisfies the following differential equation when one *does not assume slow roll conditions in the sense that even though ϵ and η are still constants, but ϵ though less than unity need not be much smaller than unity and $|\eta|$ can even be of $\mathcal{O}(1)$* (See [23])⁹ :

$$y^2(1 - \epsilon)^2 u_k''(y) + 2y\epsilon(\epsilon - \tilde{\eta}) u_k'(y) + \left(y^2 - 2 \left(1 + \epsilon - \frac{3}{2} \tilde{\eta} + \epsilon^2 - 2\epsilon\tilde{\eta} + \frac{\tilde{\eta}^2}{2} + \frac{\xi^2}{2} \right) \right) u_k(y) = 0. \tag{37}$$

In this section, following [23], we would be working with $\tilde{\eta} \equiv \eta - \epsilon$ instead of η . We will be assuming that the slow parameter $\xi \ll \ll 1$ ¹⁰ - this can be easily relaxed. In order to get a the required Minkowskian

⁹In this section, unlike subsection 3.2, to simplify calculations, we would be assuming that one continues to remain close to the locus $\sin(nk.b + mk.c) = 0$ implying that the axionic moduli space metric is approximately a constant and the axionic kinetic terms, and in particular the inflaton kinetic term, with a proper choice of basis - see [2] - can be cast into a diagonal form. The cases having to do with being away from the slow-roll scenarios are effected by an appropriate choice of the holomorphic isometric involution involved in the Swiss-Cheese Calabi-Yau orientifold and (30)

¹⁰From [23], $\xi^2 = \epsilon\tilde{\eta} - \epsilon \frac{d\tilde{\eta}}{d\mathcal{I}}$, \mathcal{I} being the inflaton of section 3.1 . Neglecting ξ can be effected if $\tilde{\eta} \sim e^{\int \sqrt{\epsilon} d\mathcal{I}}$ - this is hence appropriate for hybrid inflationary scenarios, which is what gets picked out to ensure that the curvature perturbations do not grow at horizon crossing and beyond.

free-field solution in the long wavelength limit - the following is the solution¹¹:

$$u_k(y) \sim c(k)y^{\frac{1-\epsilon^2+2\epsilon(-1+\tilde{\eta})}{2(-1+\epsilon)^2}} H^{(2)}_{\frac{\sqrt{9+9\epsilon^4-12\tilde{\eta}+4\tilde{\eta}^2-4\epsilon^3(1+5\tilde{\eta})-4\epsilon(3-3\tilde{\eta}+2\tilde{\eta}^2)+2\epsilon^2(1+6\tilde{\eta}+4\tilde{\eta}^2)}}{2(-1+\epsilon)^2}} \left(\frac{y}{(-1+\epsilon)} \right). \quad (38)$$

Now, $H_\alpha^{(2)} \equiv J_\alpha - i \left(\frac{J_\alpha \cos(\alpha\pi) - J_{-\alpha}}{\sin(\alpha\pi)} \right)$.¹² The power spectrum of scalar perturbations is then given by:

$$P_R^{\frac{1}{2}}(k) \sim \left| \frac{u_k(y=1)}{z} \right| \sim \left| H_{\tilde{\nu}}^{(2)} \left(\frac{1}{(\epsilon-1)} \right) \right| \frac{1}{\sqrt{\epsilon}}, \quad (41)$$

where $z \sim a\sqrt{\epsilon}$ (in $M_\pi = 1$ units), and

$$\tilde{\nu} \equiv \frac{\sqrt{9+9\epsilon^4-12\tilde{\eta}+4\tilde{\eta}^2-4\epsilon^3(1+5\tilde{\eta})-4\epsilon(3-3\tilde{\eta}+2\tilde{\eta}^2)+2\epsilon^2(1+6\tilde{\eta}+4\tilde{\eta}^2)}}{2(-1+\epsilon)^2}.$$

The tensor perturbation modes $v_k(y)$ satisfy the following equation (See [23]):

$$y^2(1-\epsilon)^2 v_k''(y) + 2y\epsilon(\epsilon-\tilde{\eta})u_k'(y) + (y^2 - (2-\epsilon))v_k(y) = 0. \quad (42)$$

Using arguments similar to ones given for scalar perturbation modes' solution, one can show that the solution to (42) is given by:

$$v_k(y) \sim y^{\frac{1-\epsilon^2+2\epsilon(-1+\tilde{\eta})}{2(-1+\epsilon)^2}} H^{(2)}_{\frac{\sqrt{9+\epsilon^4+4\epsilon(-6+\tilde{\eta})-4\epsilon^3\tilde{\eta}+2\epsilon^3(9-4\tilde{\eta}+2\tilde{\eta}^2)}}{2(-1+\epsilon)^2}} \left(\frac{y}{(\epsilon-1)} \right). \quad (43)$$

¹¹We follow [24] and hence choose $H_\nu^{(2)}(\frac{y}{-1+\epsilon})$ as opposed to $H_\nu^{(1)}(-\frac{y}{-1+\epsilon})$

¹²One would be interested in taking the small-argument limit of the Bessel function. However, the condition for doing the same, namely $0 < \left| \frac{y}{-1+\epsilon} \right| \ll \sqrt{\nu+1}$ is never really satisfied. One can analytically continue the Bessel function by using the fact that $J_{\tilde{\nu}}(\frac{y}{-1+\epsilon})$ can be related to the Hypergeometric function ${}_0F_1\left(\tilde{\nu}+1; -\frac{y^2}{4(1-\epsilon)^2}\right)$ as follows:

$$J_{\tilde{\nu}}\left(\frac{y}{(1-\epsilon)}\right) = \frac{\left(\frac{y}{2(1-\epsilon)}\right)^{\tilde{\nu}}}{\Gamma(\tilde{\nu}+1)} {}_0F_1\left(\tilde{\nu}+1; -\frac{y^2}{4(1-\epsilon)^2}\right). \quad (39)$$

where $\tilde{\nu} \equiv \frac{\sqrt{9+9\epsilon^4-12\tilde{\eta}+4\tilde{\eta}^2-4\epsilon^3(1+5\tilde{\eta})-4\epsilon(3-3\tilde{\eta}+2\tilde{\eta}^2)+2\epsilon^2(1+6\tilde{\eta}+4\tilde{\eta}^2)}}{2(-1+\epsilon)^2}$. Now, the small-argument limit of (38) can be taken only if

$$\left| \frac{y}{2(1-\epsilon)} \right| < 1. \quad (40)$$

This coupled with the fact that $\epsilon < 1$ for inflation - see [23] - and that (40) will still be satisfied at $y = 1$ - the horizon crossing - tells us that $\epsilon < 0.5$. One can in fact, retain the $\left(\frac{y}{-1+\epsilon}\right)^{-a}$ prefactor for continuing beyond $\epsilon = 0.5$ up to $\epsilon = 1$, by using the following identity that helps in the analytic continuation of ${}_0F_1(a; z)$ to regions $|z| > 1$ (i.e. beyond (40))- see [53]:

$$\begin{aligned} \frac{{}_0F_1(a; z)}{\Gamma(a)} &= -\frac{e^{\frac{i\pi}{2}\left(\frac{3}{2}-a\right)} z^{\frac{1-2a}{4}}}{\sqrt{\pi}} \left[\sinh\left(\frac{\pi i}{2}\left(\frac{3}{2}-a\right) - 2\sqrt{z}\right) \sum_{k=0}^{\left[\frac{1}{4}(2|b-1|-1)\right]} \frac{(2k+|a-1|-\frac{1}{2})!}{2^{4k}(2k)! (|a-1|-2k-\frac{1}{2})! z^k} \right. \\ &\quad \left. + \frac{1}{\sqrt{z}} \cosh\left(\frac{\pi i}{2}\left(\frac{3}{2}-a\right) - 2\sqrt{z}\right) \sum_{k=0}^{\left[\frac{1}{4}(2|b-1|-1)\right]} \frac{(2k+|a-1|-\frac{1}{2})!}{2^{4k}(2k+1)! (|a-1|-2k-\frac{1}{2})! z^k} \right], \end{aligned}$$

if $a - \frac{1}{2} \in \mathbf{Z}$.

The power spectrum for tensor perturbations is given by:

$$P_g^{\frac{1}{2}}(k) \sim |v_k(y=1)| \sim \left| H_{\tilde{\nu}}^{(2)} \left(\frac{1}{-1+\epsilon} \right) \right|, \quad (44)$$

where $\tilde{\nu} \equiv \frac{\sqrt{9+\epsilon^4+4\epsilon(-6+\tilde{\eta})-4\epsilon^3\tilde{\eta}+2\epsilon^3(9-4\tilde{\eta}+2\tilde{\eta}^2)}}{2(-1+\epsilon)^2}$. Hence, the ratio of the power spectra of tensor to scalar perturbations will be given by:

$$r \equiv \left(\frac{P_g^{\frac{1}{2}}(k)}{P_R^{\frac{1}{2}}(k)} \right)^2 \sim \epsilon \left| \frac{H_{\tilde{\nu}}^{(2)} \left(\frac{1}{(-1+\epsilon)} \right)}{H_{\tilde{\nu}}^{(2)} \left(\frac{1}{(-1+\epsilon)} \right)} \right|^2, \quad (45)$$

which, for $\epsilon = 0.0034$, $\tilde{\eta} \sim 10^{-5}$ - a set of values which are realized with Calabi-Yau volume $\mathcal{V} \sim 10^5$ and $D3$ -instanton number $n^s = 10$ for obtaining $f_{NL} \sim 10^{-2}$ and are also consistent with “freeze-out” of curvature perturbations at superhorizon scales (See (47)) - yields $r = 0.003$. One can therefore get a ratio of tensor to scalar perturbations of $\mathcal{O}(10^{-2})$ in slow-roll inflationary scenarios in Swiss-Cheese compactifications. Further, one sees that the aforementioned choice of ϵ and $\tilde{\eta}$ implies choosing the holomorphic isometric involution as part of the Swiss-Cheese Calabi-Yau orientifolding, is such that the maximum degree of the genus-0 holomorphic curve to be such that $n_\beta^0 \sim \frac{\mathcal{V}}{k^2 g_s^{\frac{3}{2}}}$, which can yield the number of e-foldings $N_e \sim \mathcal{O}(10)$ for $D3$ -instanton number $n^s \sim 10$ alongwith the non-Gaussianities parameter $f_{NL} \sim \mathcal{O}(10^{-2})$ and tensor-to scalar ratio $r = 0.003$.

The expression for the scalar Power Spectrum at the super horizon scales i.e. near $y = 0$ with $a(y)H(y) = \text{constant}$, is given as:

$$P_R^{(\frac{1}{2})}(y) \sim \frac{(1-\epsilon)^{\tilde{\nu}} y^{\frac{3}{2}-\tilde{\nu}}}{H^{\nu-\frac{3}{2}} a^{\nu-\frac{1}{2}} \sqrt{\epsilon}} \sim A H(y) y^{\frac{3}{2}-\tilde{\nu}} \quad (46)$$

where $\nu = \frac{1-\epsilon^2+2\epsilon(-1+\tilde{\eta})}{2(-1+\epsilon)^2}$ and A is some scale invariant quantity. Using $\frac{d \ln H(y)}{d \ln y} \equiv \frac{\epsilon}{1-\epsilon}$, we can see that scalar power spectrum will be frozen at superhorizon scales, i.e., $\frac{d \ln P^{\frac{1}{2}}(y)}{d \ln y} = 0$ if the allowed values of ϵ and $\tilde{\eta}$ parameters satisfy the following constraint:

$$\frac{d \ln H(y)}{d \ln y} + \frac{3}{2} - \tilde{\nu} \equiv \frac{\epsilon}{1-\epsilon} + \frac{3}{2} - \tilde{\nu} \sim 0 \quad (47)$$

The loss of scale invariance is parameterized in terms of the spectral index which is:

$$n_R - 1 \equiv \frac{d \ln P(k)}{d \ln k} = 3 - 2 \text{Re}(\tilde{\nu}) \quad (48)$$

which gives the value of spectral index $n_R - 1 = 0.014$ for the allowed values e.g. say ($\epsilon = 0.0034$, $\tilde{\eta} = 0.000034$) obtained with curvature fluctuations frozen of the order 10^{-2} at super horizon scales.

In a nutshell, for $\mathcal{V} \sim 10^5$ and $n^s \sim 10$ we have $\epsilon \sim 0.0034$, $|\eta| \sim 0.000034$, $N_e \sim 17$, $|f_{NL}|_{\max} \sim 10^{-2}$, $r \sim 4 \times 10^{-3}$ and $|n_R - 1| \sim 0.014$ with super-horizon freezout condition's violation of $\mathcal{O}(10^{-3})$. Further if we try to satisfy the freeze-out condition more accurately, say we take the deviation from zero of the RHS of (47) to be of $\mathcal{O}(10^{-4})$ then the respective set of values are: $\mathcal{V} \sim 10^6$, $n^s \sim 10$, $\epsilon \sim 0.00028$, $|\eta| \sim 10^{-6}$, $N_e \sim 60$, $|f_{NL}|_{\max} \sim 0.01$, $r \sim 0.0003$ and $|n_R - 1| \sim 0.001$. This way, we have realized $N_e \sim 60$, $f_{NL} \sim 10^{-2}$, $r \sim 10^{-3}$ and an almost scale-invariant spectrum in the slow-roll case of our LVS Swiss-Cheese Calabi-Yau orientifold setup.

5 Conclusion and Discussion

In this note, we argued that starting from large volume compactification of type IIB string theory involving orientifolds of a two-parameter Swiss-Cheese Calabi-Yau three-fold, for appropriate choice of the holomorphic isometric involution as part of the orientifolding and hence the associated Gopakumar-Vafa invariants corresponding to the maximum degrees of the genus-zero rational curves, it is possible to obtain f_{NL} - parameterizing non-Gaussianities in curvature perturbations - to be of $\mathcal{O}(10^{-2})$ in slow-roll and to be of $\mathcal{O}(1)$ in slow-roll violating scenarios. Using general considerations and some algebraic geometric assumptions as above, we show that requiring a “freezeout” of curvature perturbations at super horizon scales, it is possible to get tensor-scalar ratio of $\mathcal{O}(10^{-3})$ in the same slow-roll Swiss-Cheese setup. We predict loss of scale invariance to be within the existing experimental bounds. In a nutshell, for Calabi-Yau volume $\mathcal{V} \sim 10^6$ and $n^s \sim 10$, we have realized $\epsilon \sim 0.00028$, $|\eta| \sim 10^{-6}$, $N_e \sim 60$, $|f_{NL}|_{max} \sim 0.01$, $r \sim 0.0003$ and $|n_R - 1| \sim 0.001$ with a super-horizon-freezeout condition's deviation (from zero) of $\mathcal{O}(10^{-4})$. Further we can see that with Calabi-Yau volume $\mathcal{V} \sim 10^5$ and $n^s \sim 10$ one can realize better values of non-Gaussianities parameter and “r” ratio ($|f_{NL}|_{max} = 0.03$ and $r = 0.003$) but with number of e-foldings less than 60. Also in the slow-roll violating scenarios, we have realized $f_{NL} \sim \mathcal{O}(1)$ with number of e-foldings $N_e \sim 60$ without worrying about the tensor-to-scalar ratio and $|n_R - 1|$ parameter.

To conclude, we would like to make some curious observations pertaining to the intriguing possibility of dark matter being modelled by the NS-NS axions presenting the interesting scenario of unification of inflation and dark matter and producing finite values of non-Gaussianities and tensor-scalar ratio. In (2), if one assumes:

- (a) the degrees k_a 's of $\beta \in H_2^-(CY_3)$ are such that they are very close and large which can be quantified as $\frac{k_1^2 - k_2^2}{k_1^2 + k_2^2} \sim -\mathcal{O}\left(\frac{1}{2\sqrt{\ln \mathcal{V} \mathcal{V}^4}}\right)$,
 - (b) one is close to the locus $\sin(nk.b + mk.c) = 0$, where the closeness is quantified as $\sin(nk.b + mk.c) \sim \mathcal{O}(\frac{1}{\mathcal{V}})$, and
 - (c) the axions are sub-Planckian so that one can disregard quadratic terms in axions relative to terms linear in the same,
- then the potential of (2) can then be written as

$$V \sim V_0 \left(\left(\sum_{m^a} e^{-\frac{m^2}{2g_s} + \frac{m_a b^a n^s}{g_s}} \right)^2 - 8 \right). \quad (49)$$

Now, the Jacobi theta function ($\theta(\frac{i}{g_s}, \frac{n^s b^a}{g_s})$) squared in (49) can be rewritten as:

$$\sum_{\mathcal{M}_1^+, \mathcal{M}_1^{+-}; \mathcal{M}_2^+, \mathcal{M}_2^-} e^{-\frac{(\mathcal{M}_1^+)^2 + (\mathcal{M}_1^-)^2}{2g_s}} e^{-\frac{(\mathcal{M}_2^+)^2 + (\mathcal{M}_2^-)^2}{2g_s}} e^{(\mathcal{M}_1^+ b^1 + \mathcal{M}_2^+ b^2) \frac{n^s}{g_s}}. \quad (50)$$

Now, writing $m_1 b^1 + m_2 b^2$ as $\frac{1}{2}(\mathcal{M}_+(b^1 + b^2) + \mathcal{M}_-(b^1 - b^2))$ and noting that the inflaton \mathcal{I} , for $k_1 \sim k_2$ can be identified with $b^1 + b^2$ - see [2] - one sees that (50) can be written as

$$\left(\sum_{\mathcal{M}_1^-} e^{-\frac{(\mathcal{M}_1^-)^2}{4g_s}} \right)^2 \sum_{\mathcal{M}_+, \mathcal{M}_-} e^{-\frac{(\mathcal{M}_+)^2 + (\mathcal{M}_-)^2}{2g_s}} e^{\frac{(\mathcal{M}_+ \mathcal{I} + \mathcal{M}_- \mathcal{I}^\perp) n^s}{2}}, \quad (51)$$

$\mathcal{I}^\perp \sim b^1 - b^2$ - for orthonormal axionic fields, \mathcal{I}^\perp will be orthogonal to \mathcal{I} . Now, assuming \mathcal{I}^\perp has been

stabilized to 0, one sees that one could write

$$\left(\theta\left(\frac{i}{g_s}, \frac{b^a n^s}{g_s}\right)\right)^2 \sim 2 \left(\sum_{\mathcal{M}_1^-} e^{-\frac{(\mathcal{M}_1^-)^2}{2g_s}}\right)^3 \sum_{\mathcal{M}_+ \geq 0} e^{-\frac{\mathcal{M}_+^2}{2g_s}} \cosh\left(\frac{\mathcal{M}_+ \mathcal{I} n^s}{2}\right), \quad (52)$$

which in the weak coupling limit $g_s \ll 1$ is approximately equal to $2 \sum_{\mathcal{M}_1 \geq 0} e^{-\frac{\mathcal{M}_1^2}{4g_s}} \cosh\left(\frac{\mathcal{M}_+ \mathcal{I} n^s}{2}\right)$. This, when substituted into the expression for the potential in (49), yields:

$$V \sim V_0 \left(\sum_{\mathcal{M} \geq 0} e^{-\frac{\mathcal{M}^2}{2g_s}} \cosh\left(\frac{\mathcal{M} \mathcal{I} n^s}{2}\right) - \sum_{\mathcal{M} \geq 0} e^{-\frac{\mathcal{M}^2}{2g_s}} \right). \quad (53)$$

Once again, in the weak coupling limit, the sum in (53) can be assumed to be restricted to \mathcal{M} proportional to 0 and 1. This hence gives:

$$V \sim V_0 \left(\cosh\left(\frac{\mathcal{I} n^s}{2}\right) - 1 \right). \quad (54)$$

One sees that (54) is of the same form as the potential proposed in [56]:

$$V = V_0 (\cosh(\lambda\phi) - 1),$$

for cold dark matter! This, given the assumption of sub-Planckian axions, is by no means valid for all \mathcal{I} . However, in the given domain of validity, the fact that a string (SUGRA) potential can be recast into the form (54) is, we feel, quite interesting.

Alternatively, in the same spirit as [57], if one breaks the NS-NS axionic shift symmetry “slightly” by restricting the symmetry group \mathbf{Z} to $\mathbf{Z}_+ \cup \{0\}$, then (53) can be rewritten as:

$$\sum_{\mathcal{M} \geq 0} e^{-\frac{\mathcal{M}^2}{2g_s}} e^{\frac{\mathcal{M} \mathcal{I} n^s}{2}} - \sum_{\mathcal{M} \geq 0} e^{-\frac{\mathcal{M}^2}{2g_s}} \sim e^{-\frac{\pi^2}{2g_s}} e^{\frac{\pi \mathcal{I} n^s}{2}} + e^{-\frac{4\pi^2}{2g_s}} e^{\frac{4\pi \mathcal{I} n^s}{2}}, \quad (55)$$

which is similar to:

$$V = e^{\alpha_1 + \alpha_2 \phi} + e^{\beta_1 + \beta_2 \phi},$$

(where α_2, β_2 are taken to be positive) that has been used to study quintessence models (in studies of dark energy) - see [58] - in fact, as argued in [58], one can even include \mathcal{I}^\perp .

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